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Mathematics and Students with Down Syndrome: A Vision for Success

*National Down Syndrome World Congress
51st Annual Convention*

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WELCOME!

- Introductions
- Handout: Mathematics Instruction Planning and Implementation Guides for Students with Down Syndrome
- A Vision of What Mathematics Can Be for Students with DS
- Individuals with Down Syndrome: A Framework for Success
- Putting the Framework Into Action: Examples
- Conversation



Dr. Lauer



Dr. Allsopp

<https://www.youtube.com/watch?v=YoTlaRyGzac>

<https://www.youtube.com/watch?v=IzxVyO6cpos>

The Problems of Math

Vision

What is Your Vision for What Math Can Be for Students with Down Syndrome?

- What is Mathematics?
- What does it mean to “Do Math?”
- What does it mean to be “successful” in Math?



Reflect & Share

Let's Expand Our Vision!



A Literacy Analogy

- Phonics, site words, reading comprehension: all pieces to the literacy puzzle.
- The goal is not the “th” sound or 30 words per minute.
- The goal is to read fluently because reading is how we know we are not alone.
- As we learn to read and write, we gain the ability to be connected to others- to learn from them and to express ourselves.
- That HUMAN connection is the point, and we want our children to find PLEASURE AND JOY in reading.



Expanded Vision for Mathematics

“As a practical matter, **mathematics is a science of pattern and order.**”

Everybody Counts: Report to the Nation on the Future of Mathematics Ed. ©1989
by National Academy of Sciences, National Academy Press, Washington, D.C.

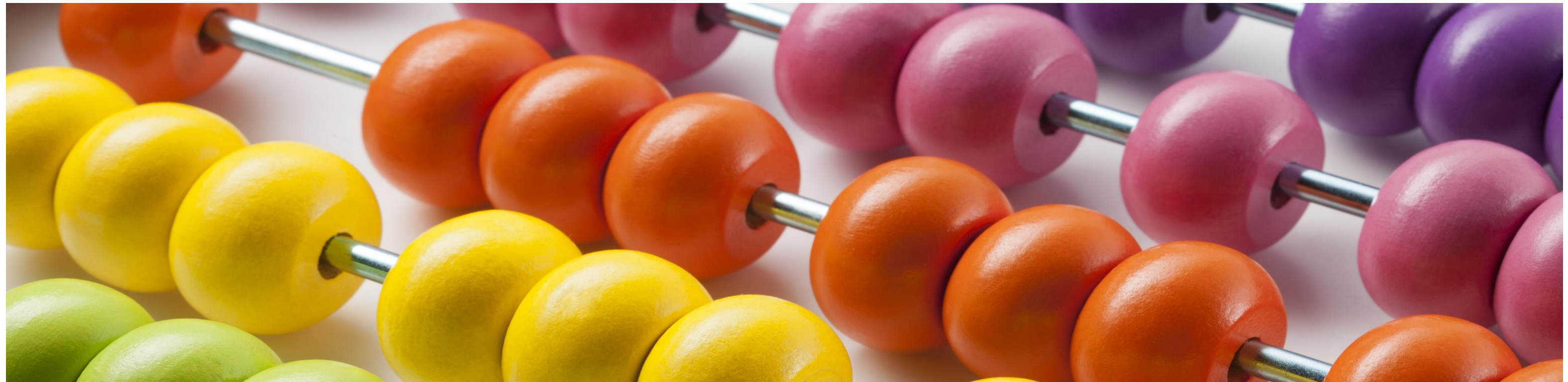
Mathematics is the language that helps us understand and explain the structure around us.

It's a way of making meaning of the world.



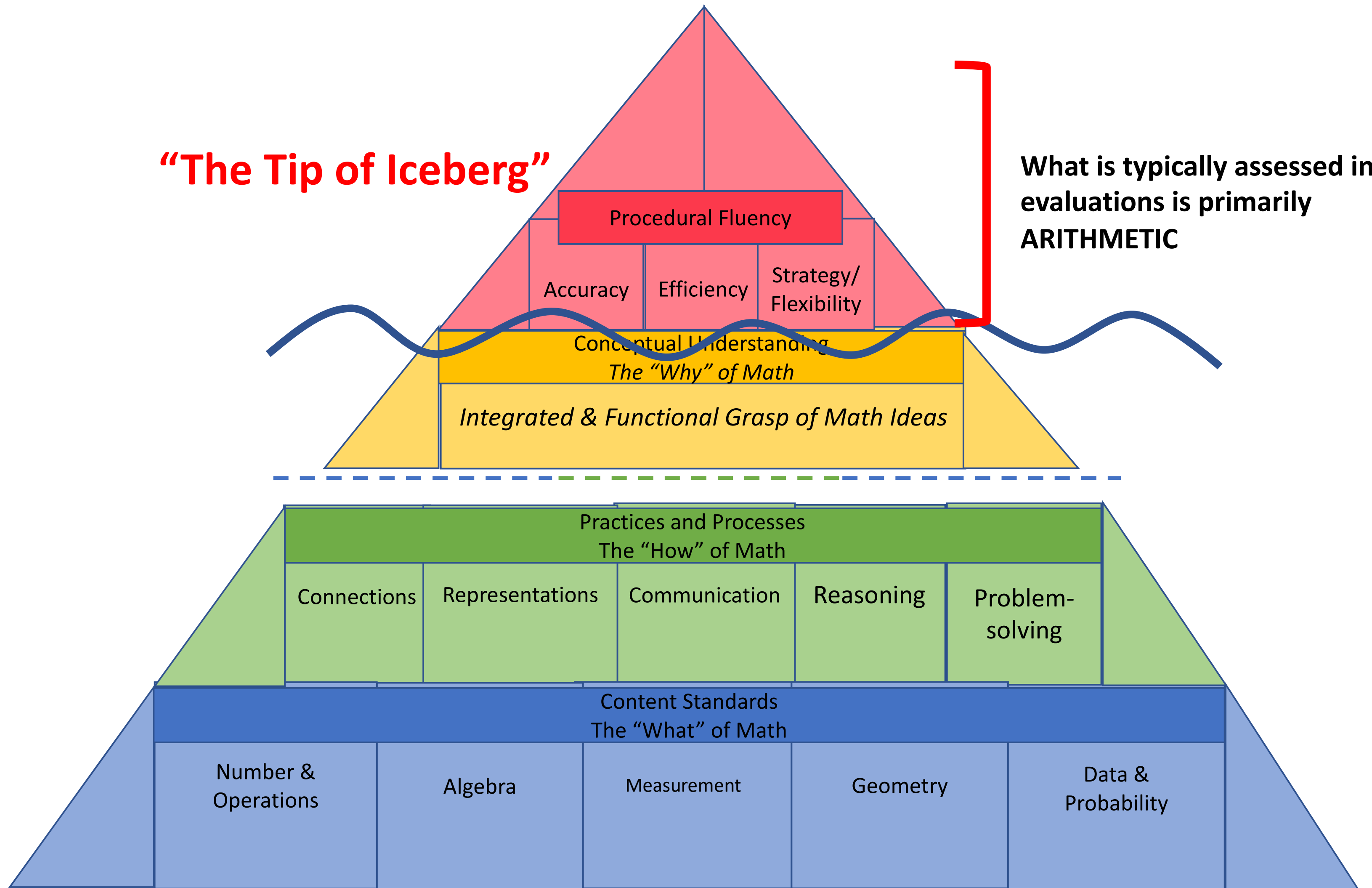
ENGAGING IN *MATHEMATICS*

Just as the end goal in reading is NOT merely decoding, but equipping the child to communicate with others, so too, the end goal in mathematics is NOT merely arithmetic (+/-/x/÷) but **recognizing and using pattern/structure to understand the world around them.**

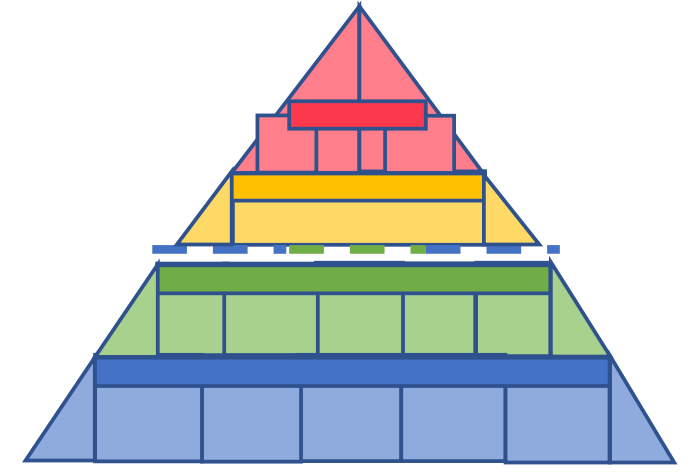


“The Tip of Iceberg”

What is typically assessed in evaluations is primarily **ARITHMETIC**

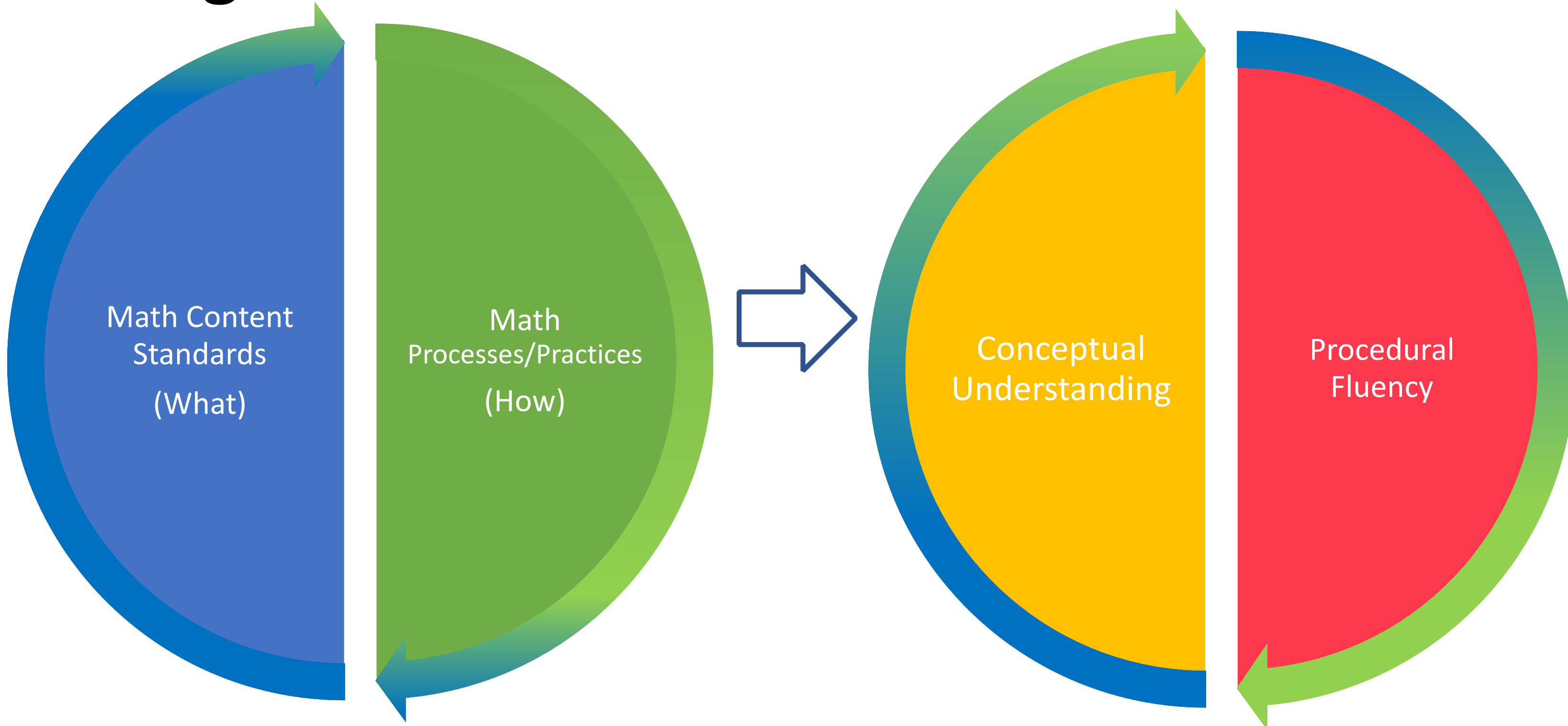


The Mathematics Pyramid



Doing Mathematics

Student Outcomes



Math

Student knows the WHY of a procedure.

Integrated & functional knowledge

Connect new ideas to previous ones- neural network

Using different representations for different purposes

Can link content and methods

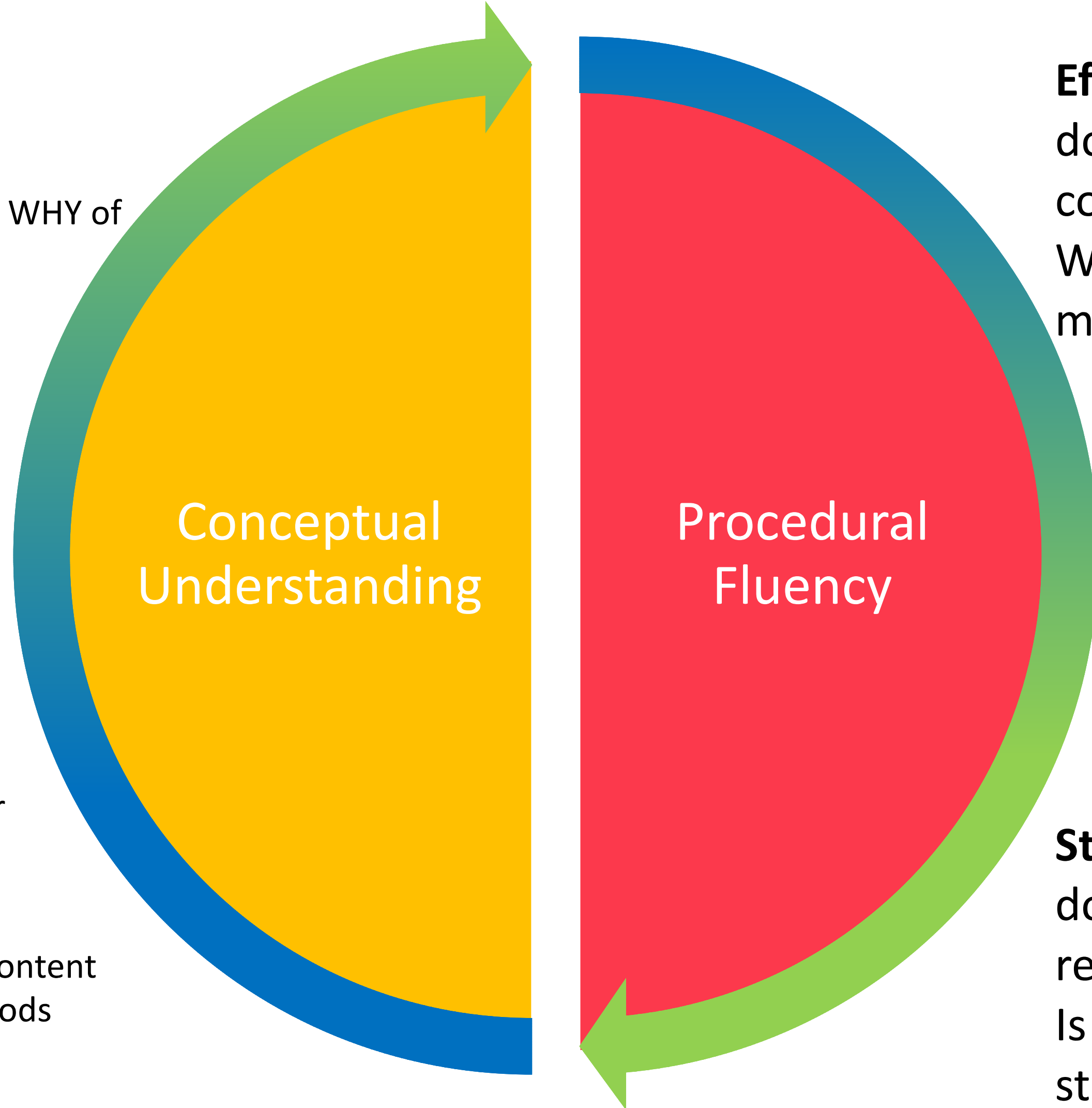
Conceptual Understanding

Procedural Fluency

Efficiency-how fast does a student complete problems? Which solution method?

Accuracy-how many problems does the student get correct?

Strategy/Flexibility-does the student recognize patterns? Is the student strategic in thinking?





A Framework for Success

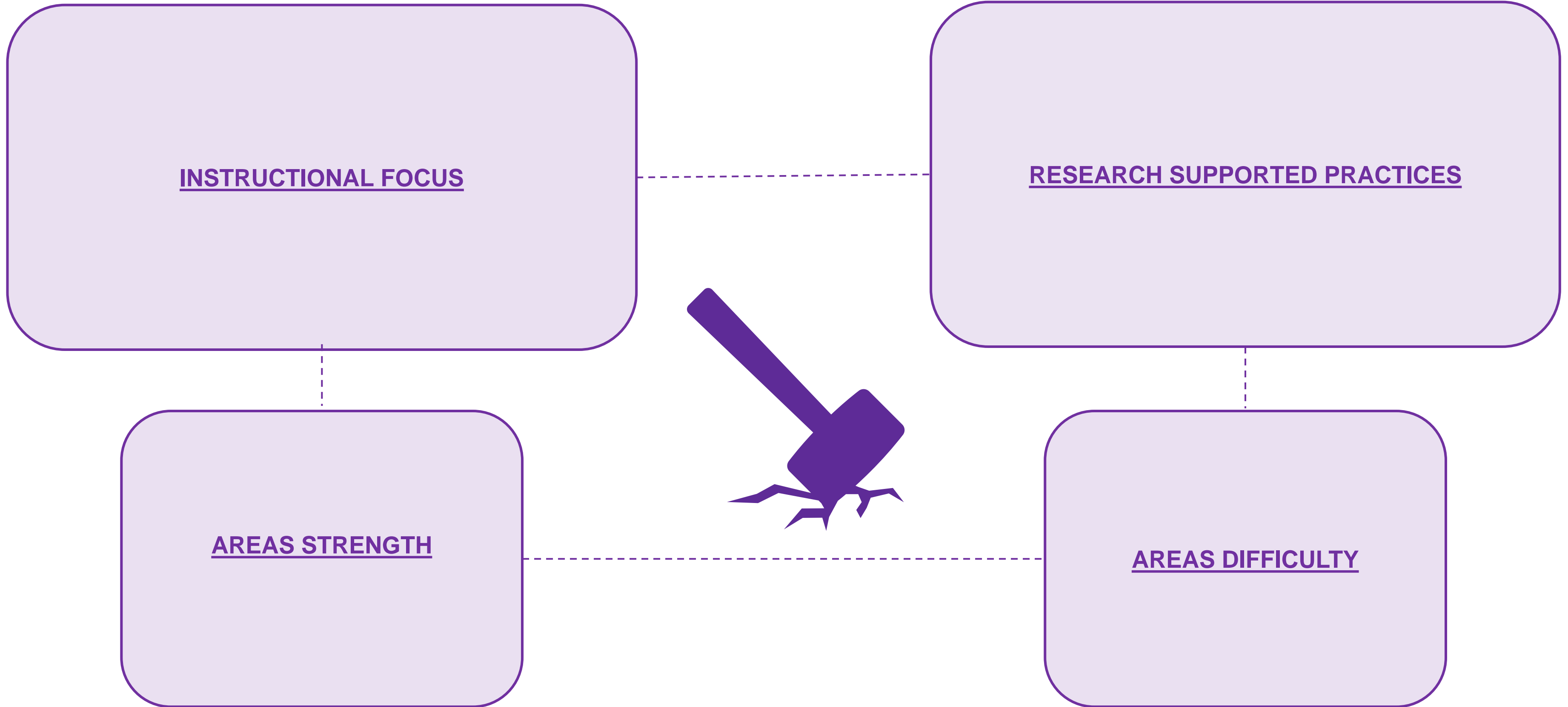


Expanded Expectations! In mathematics this looks like thinking seriously about a student's ability to recognize and use **patterns and structure**. With more expansive goals, we provide a more robust foundation for students with DS and related exceptionalities.

Choose Curiosity! Rather than becoming frustrated at lack of progress or slow progress, become curious. What could be different in this learning opportunity?

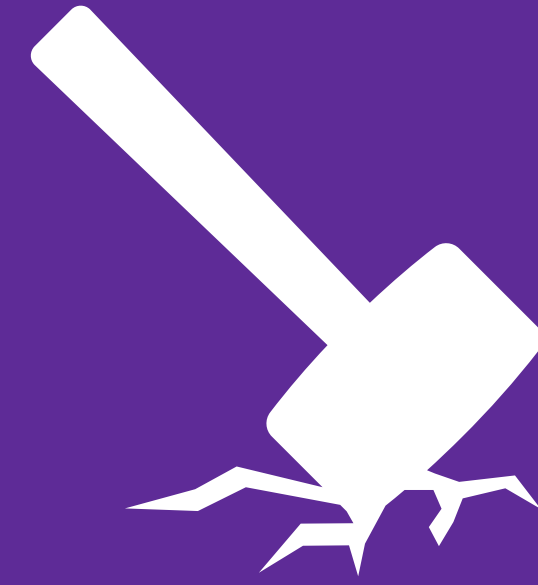
Go slow to go fast! Much of what we will do today is to talk about slowly building a strong math foundation that can sustain more success in math that you or your students/children might have thought possible.

Breaking Through the Wall Blocking Progress



Instructional Focus

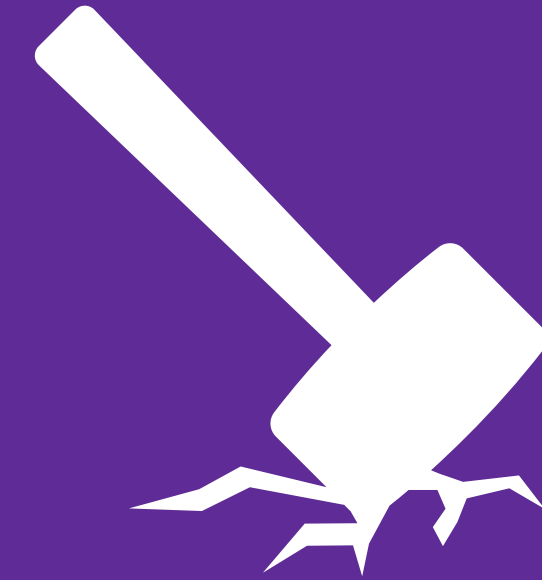
- Use Continuous Formative Assessment
- Use Coherent and Connected Mathematics Content
- Integrate Conceptual Understanding with
- Procedural Fluency
- Engage in the Mathematics
- Processes



We must widen our instructional focus

Areas of Strength

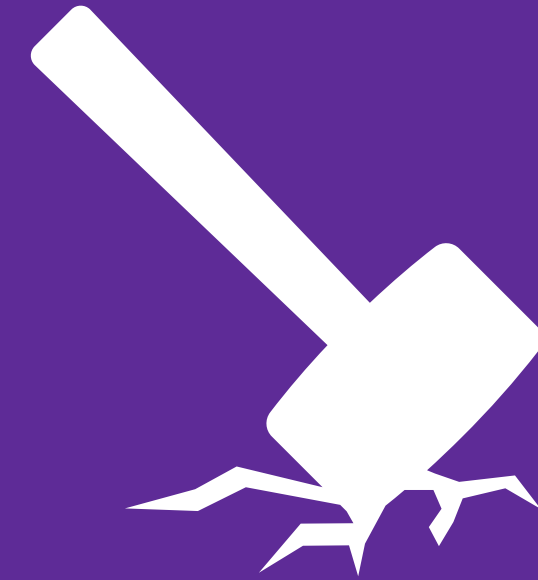
- Social Interaction
- Empathy
- Visual Inputs & Short-term Memory
- Self-help/Daily Living Skills
- Word Reading/Vocabulary Acquisition
- Technology Oriented
- Other



We must acknowledge the strengths of students with DS

Areas of Difficulty

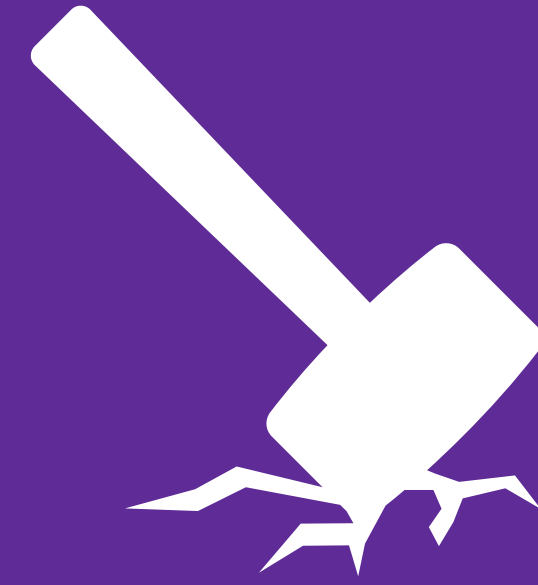
- Motor Skills (Fine)
- Expressive Language
- Verbal/Auditory Short-term Memory
- Number Skills
- Other



We must acknowledge the areas of difficulty for students with DS that impact learning mathematics

Research Supported Practices

- Peer Tutoring/Structured Cooperative Learning Groups
- Authentic Contexts
- Purposeful Use of Technology
- Game-Based Learning
- Structured Language Experiences
- Visuals
- Explicit Systematic Instruction
- Metacognitive Strategy Instruction



We must purposefully leverage the use of research-supported math instructional practices for students with exceptionalities, including students with Down Syndrome

A Framework to Achieve This Vision

INSTRUCTIONAL FOCUS

Use Continuous Formative Assessment
Use Coherent and Connected
Mathematics Content
Integrate Conceptual Understanding with
Procedural Fluency
Engage in the Mathematics
Processes

1

RESEARCH SUPPORTED PRACTICES

Peer Tutoring/Structure Collaborative Groups
Authentic Contexts
Purposeful Use of Technology
Game-Based Learning
Structured Language Experiences
Visuals
Explicit Systematic Instruction
Metacognitive Strategy Instruction

4

AREAS STRENGTH

Social Interaction
Empathy
Visual Inputs & Short-term
Memory
Self-help/Daily Living Skills
Word Reading/Vocabulary
Acquisition
Technology Oriented

2

AREAS OF DIFFICULTY

Motor Skills (Fine)
Expressive Language
Verbal/Auditory Short-term
Memory
Number Skills

3

Putting the Framework In Action: Examples

Early Grades: Math Facts (*By Leveraging Algebra/Algebraic Thinking*)

Middle/High School: Algebraic Equations (*By Leveraging Geometry*)

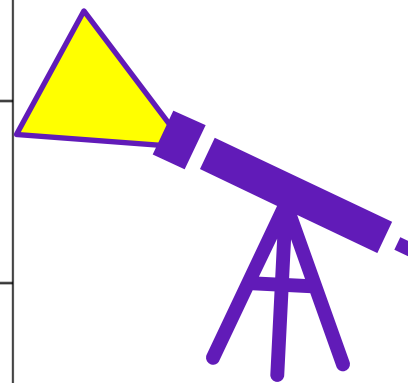
Handout: The Mathematics Instruction Planning and Implementation Guides for Students with Down Syndrome

Mathematics Instruction Planning & Implementation Guides for Students with Down Syndrome

My student's mathematics learning objective/goal is: _____

I can leverage my student's knowledge and skills in the following math content to support their success: _____

Planning Questions	Ideas to Consider	Notes
What is my learning/teaching focus?	<input type="checkbox"/> Use Continuous Formative Assessment <input type="checkbox"/> Use Coherent & Connected Math Content <input type="checkbox"/> Integrate Conceptual Understanding & Procedural Fluency <input type="checkbox"/> Engage in the Mathematics Processes	
What area(s) of difficulty might present barriers to my student's learning success given the learning/instructional focus?	<input type="checkbox"/> Motor Skills <input type="checkbox"/> Expressive Language <input type="checkbox"/> Verbal/Auditory Short-term Memory <input type="checkbox"/> Number Skills <input type="checkbox"/> Other	
What areas of strength can I leverage to promote my student's success given the learning/instructional focus?	<input type="checkbox"/> Social Interaction <input type="checkbox"/> Empathy <input type="checkbox"/> Visual Inputs & Short-term Memory <input type="checkbox"/> Self-help/Daily Living Skills <input type="checkbox"/> Word Reading/Vocabulary Acquisition <input type="checkbox"/> Technology Oriented <input type="checkbox"/> Other	
Which research-supported practice(s) can I use to promote my student's learning given the learning/instructional focus?	<input type="checkbox"/> Peer Tutoring/Structure Collaborative Groups <input type="checkbox"/> Authentic Contexts <input type="checkbox"/> Purposeful Use of Technology <input type="checkbox"/> Game-Based Learning <input type="checkbox"/> Structured Language Experiences <input type="checkbox"/> Visuals <input type="checkbox"/> Explicit Systematic Instruction <input type="checkbox"/> Metacognitive Strategy Instruction	
Which math processes/practices are best to engage my student in given the learning/instructional focus?	<input type="checkbox"/> Problem Solving <input type="checkbox"/> Reasoning and Proof <input type="checkbox"/> Communication <input type="checkbox"/> Connections <input type="checkbox"/> Representation	



Planning Guide

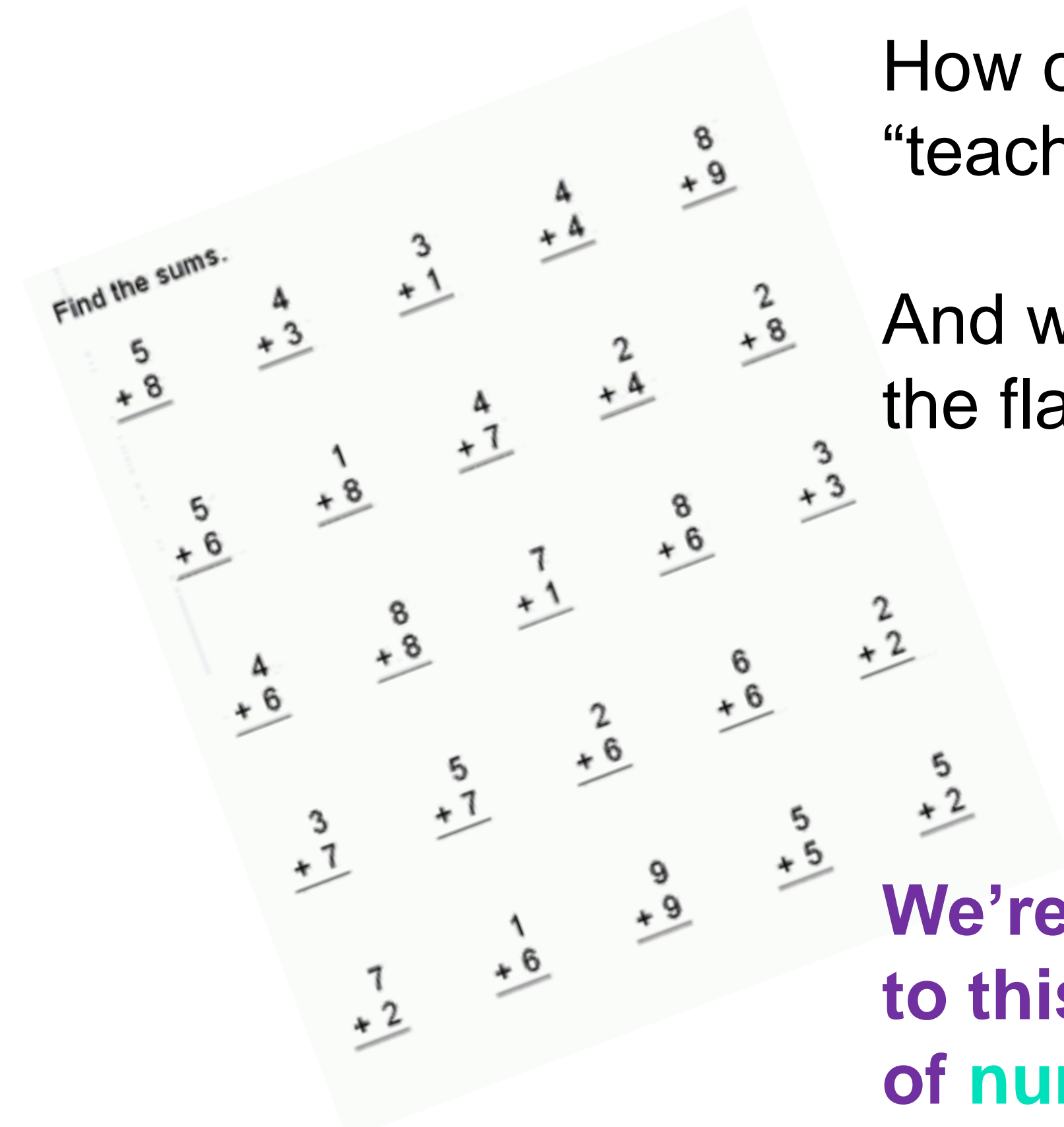
Implementation Guide

Accommodations Checklist

Early Grades: Math Facts

How often have you seen this approach to “teaching” math facts?

And when the facts pages don't work, often the flash cards come out!

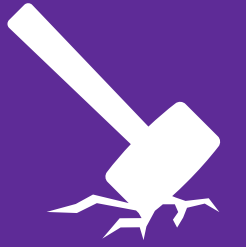


We're going to talk through an alternative to this approach that builds a foundation of **number sense** and **algebraic thinking**.



We'll start by assessing the related number sense foundation.

Instructional Focus



- Use Continuous Formative Assessment
- Use Coherent and Connected Mathematics Content
- Integrate Conceptual Understanding with Procedural Fluency
- Engage in the Mathematics Processes

Continuous Formative Assessment: Number Sense

- The skill of "Counting" is a trajectory that involves many sub-skills
- Use "dot plates" to support SUBITIZING (child's ability to immediately recognize a number of objects- like we immediately know we've rolled a five)



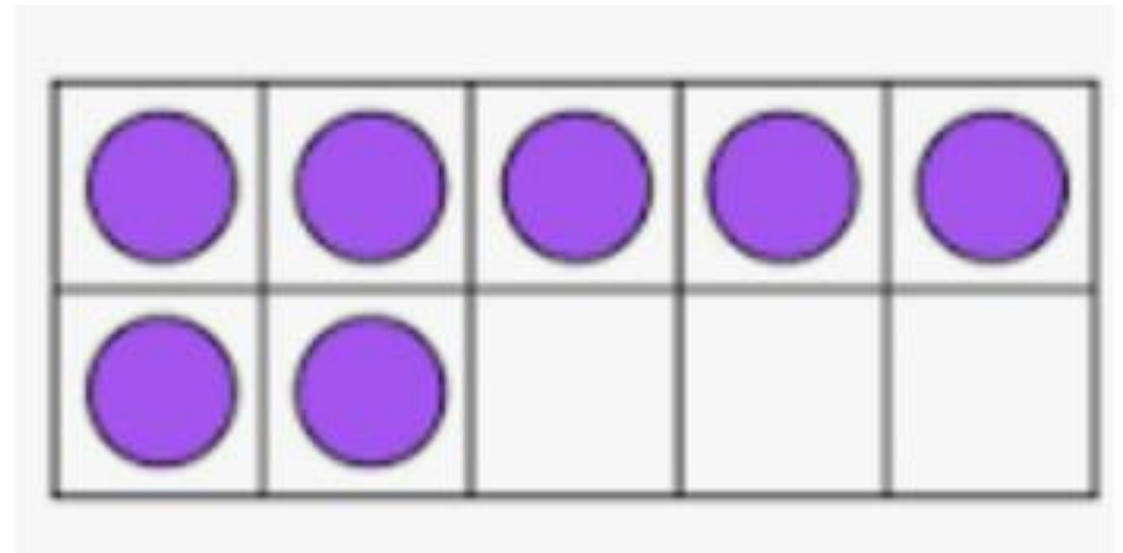
- Test for CARDINALITY (child's ability to know the last number said in a count represents the number of objects in that group)



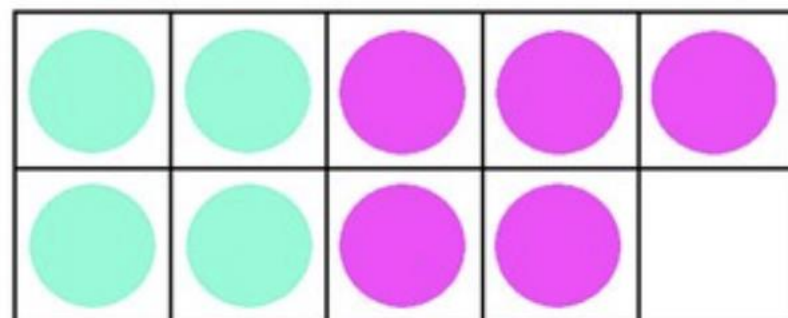
- "Please count the bears." (child counts to 6). Then ask "How many bears are in this group?" (if the child needs to recount they don't have cardinality).

Coherent & Connected Content: Number Sense

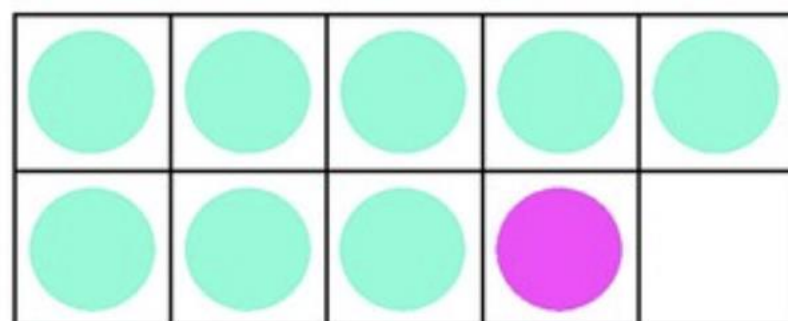
- Use 5 & 10 frames both for subitizing and for all "10 facts." You can show a frame filled with 7 dots and the students can learn to say "7 and 3 (the empty cells) makes 10."



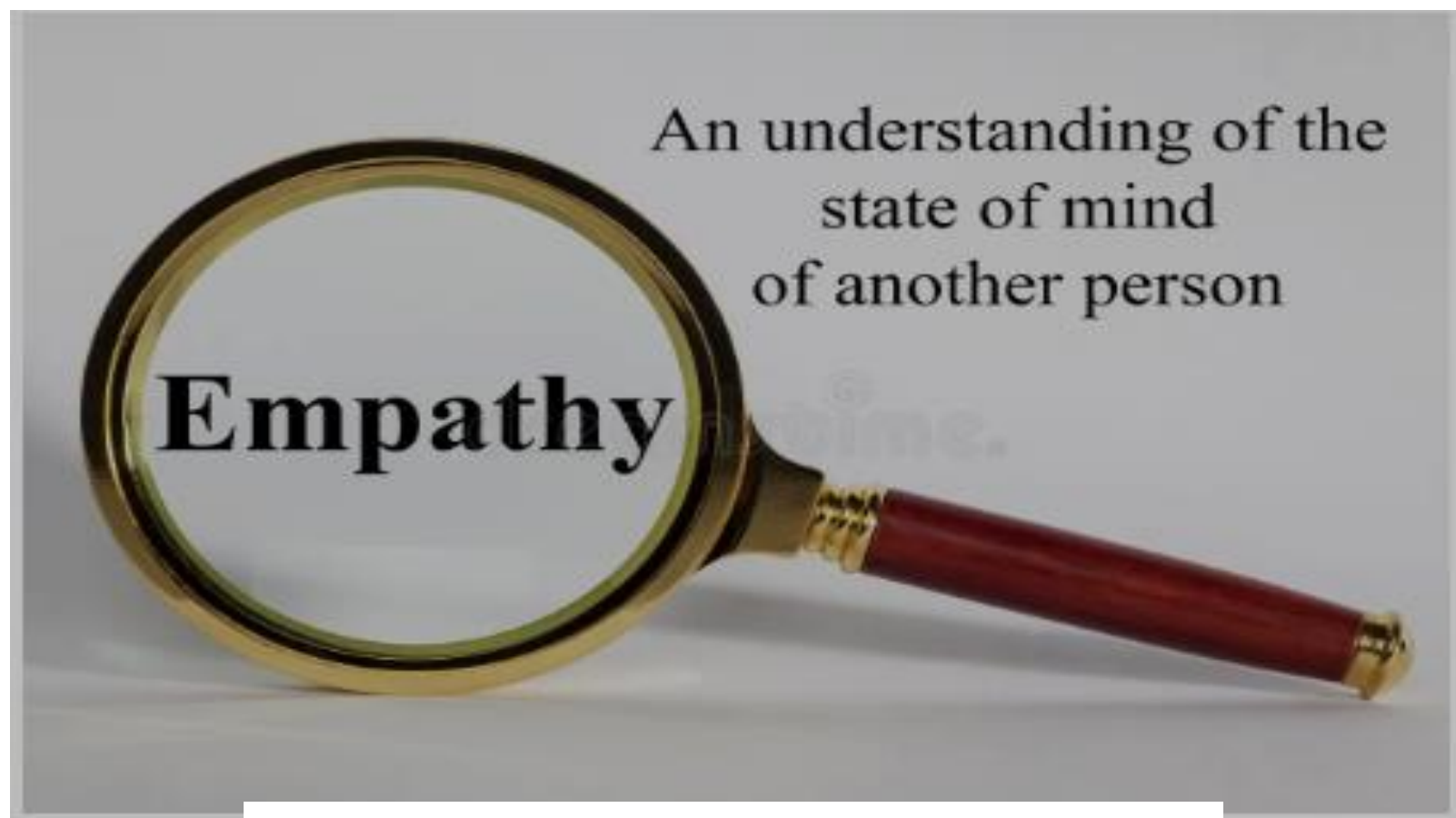
- You can also use two different color dots in a frame or two frames.



“4 and 5 make 9”



“8 and 1 make 9”



Visuals

Leverage students' personal and learning strengths

Areas of Strength



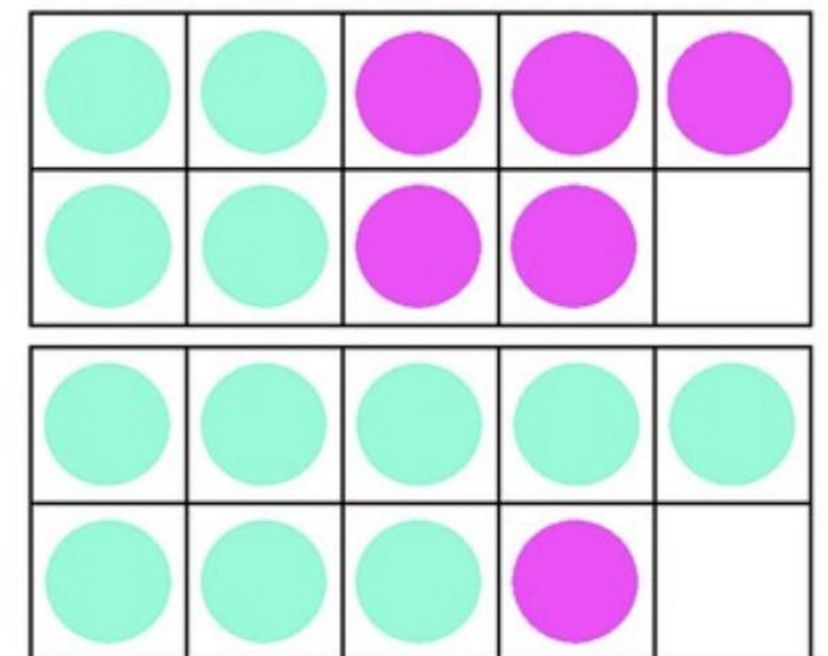
- Social Interaction
- Empathy
- Visual Inputs & Short-term Memory
- Self-help Daily Living Skills
- Word Reading/Vocabulary Acquisition
- Technology Oriented
- Other _____

Areas of Strength: Empathy and Visual Short-term Memory

- Situate the ten-frame activity within an **empathetic context**:

The student and his/her friend are at the local “sandbag fill” station helping people in their town prepare for a potential hurricane. It is hot and an elderly woman is having trouble getting the sandbags into the pull wagon she brought with her...

Challenge the student to help the woman get the number of sandbags she needs into the pull wagon using a ten-frame using different number combinations.





Accommodate Areas of Difficulty

Areas of Difficulty



- Motor Skills (Fine)
- Expressive Language
- Verbal/auditory Short-term Memory
- Number Skills
- Other _____

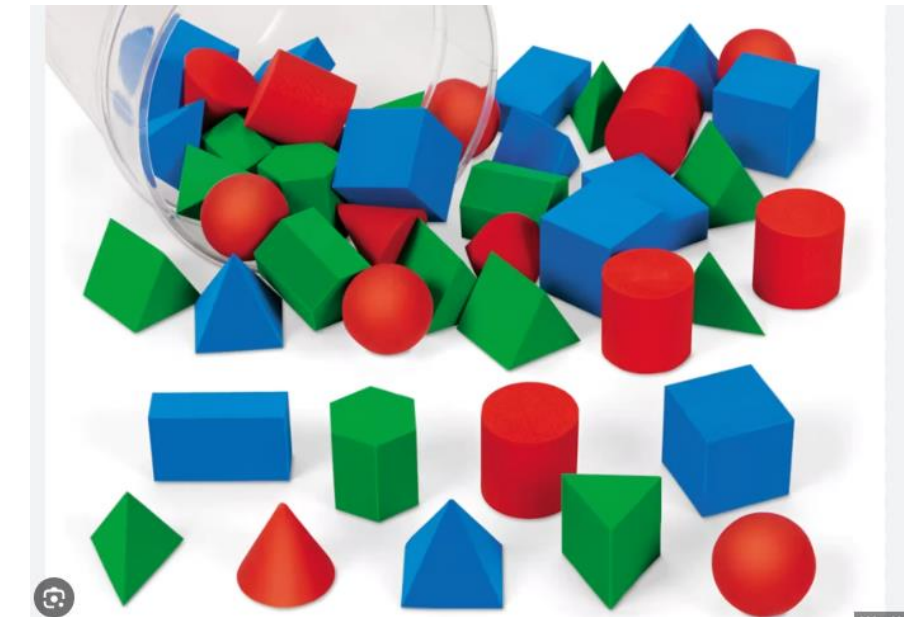
Areas of Difficulty: Motor Skills, Verbal/Auditory Short-term Memory, and Number Skills

Checklist for Considering How to Accommodate My Student's Areas of Difficulty Based on the Mathematics Learning Goal/Objective

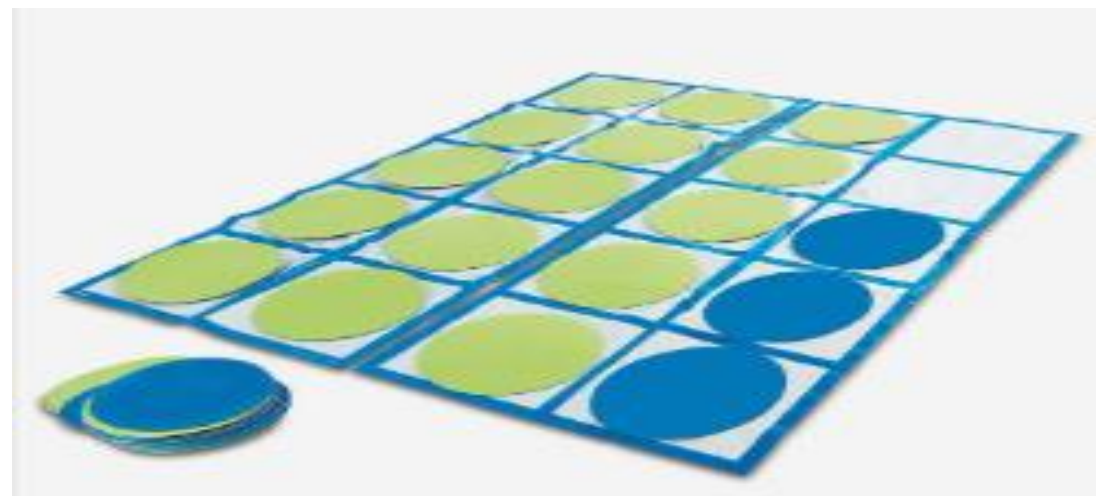
Area of Difficulty	Possible Accommodations
Motor Skills	<ul style="list-style-type: none"> <input checked="" type="checkbox"/> Provide manipulatives/concrete objects that my student can easily grasp and manipulate (e.g., larger size, foam instead of hard, etc.) <input type="checkbox"/> Make adaptations for tasks that involve the use of a pencil or pen (e.g., pencil grip, large-size pencils/pens, etc.) <input checked="" type="checkbox"/> Make space adaptations to make writing/drawing easier (e.g., highlight areas to be filled, larger spaces where responses are to be written, etc.) <ul style="list-style-type: none"> Provide <i>sentence stems</i> with blanks for writing in important words/vocabulary when engaging students in math discourse/explanations/discussions. <input type="checkbox"/> Consider technology-related accommodations that do not require written responses (teach keyboarding skills, speech-to-text software, physical mouse instead of a trackpad on a computer, etc.) <input type="checkbox"/> Other _____

Areas of Difficulty: Motor Skills, Expressive Language, Verbal/Auditory Short-term Memory, and Number Skills

Foam Manipulatives



Enlarged Ten-Frames

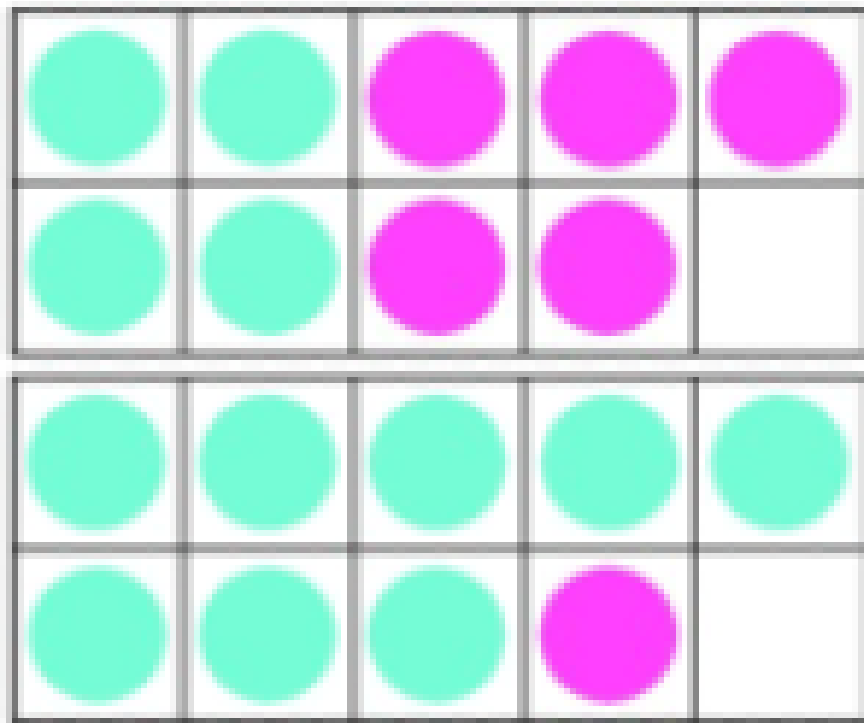


Areas of Difficulty: Motor Skills, Expressive Language, Verbal/Auditory Short-term Memory, and Number Skills

Expressive Language

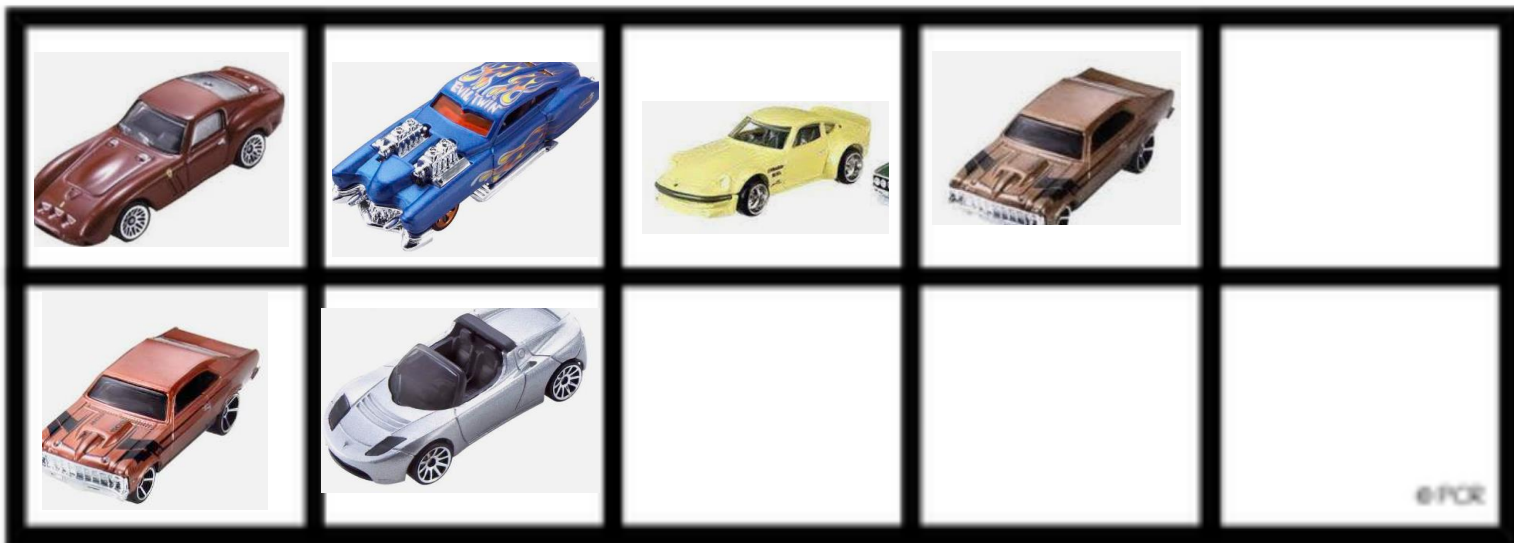
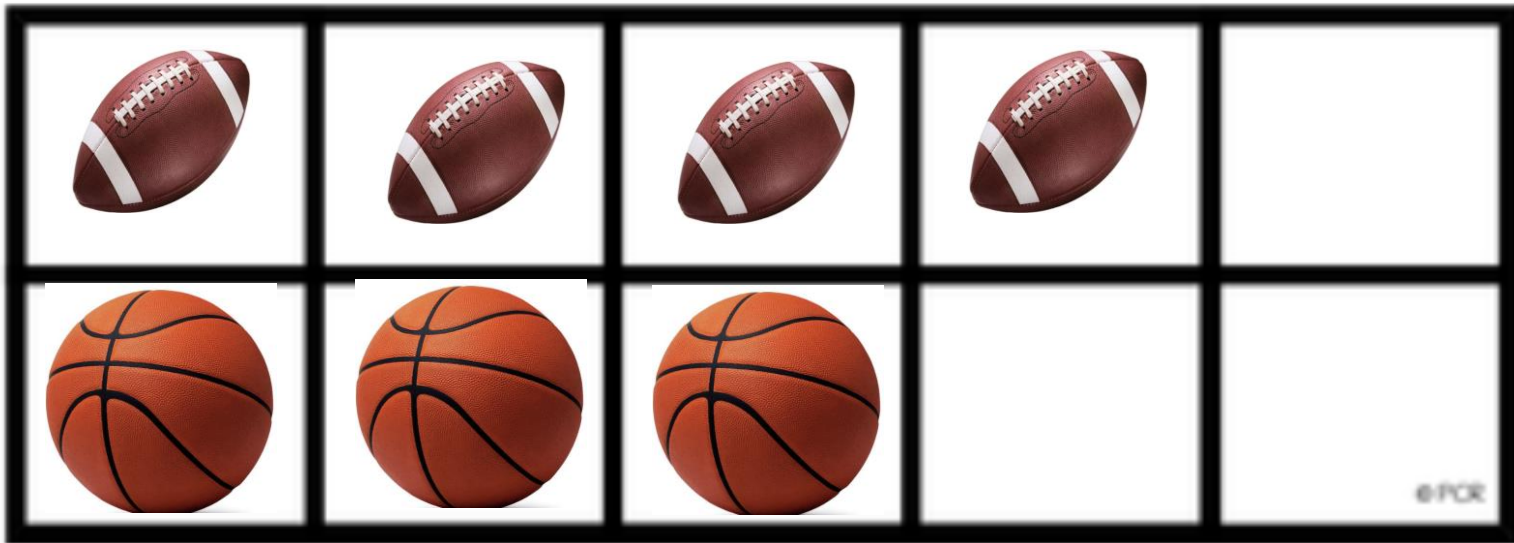
- Instead of expecting a student to initially respond verbally or in writing to an open-ended question, provide the student with choices that they can point to or select (**Tips:** *no more than 2-3 initially, ensure the difference between the correct and incorrect choice(s) is significant initially; then scaffold to a larger number of choices – 4 or 5- and differences between choices that are less significant*).
- Provide *sentence stems* with blanks for important words/vocabulary students express verbally when engaging students in math discourse/explanations/discussions (**Tip:** *provide the student with choices – see above*).
- Use computer/tablet/phone text-to-speech software.
- The teacher responds to the question/prompt and asks the student to identify the extent to which the teacher's response or explanation is accurate using a rubric or scale (e.g., yes/no; not at all/some/perfect, etc.).
- Engage the student in using manipulatives/drawings to represent their thinking.
- Other _____

Areas of Difficulty: Motor Skills, Expressive Language, Verbal/Auditory Short-term Memory, and Number Skills



Teacher: *“Point to the ten-frame that shows ‘8 and 1 make 9’.”*

What stories can we tell about these facts?



Research Supported Practices



Peer Tutoring/Structured Cooperative Groups



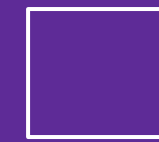
Authentic Contexts



Purposeful Use of Technology



Game-Based Learning



Structured Language Experiences



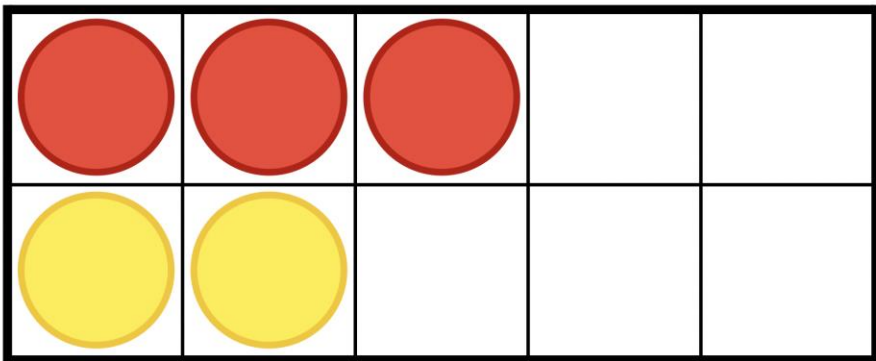
Visuals



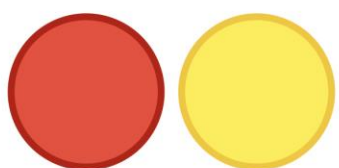
Explicit Systematic Instruction



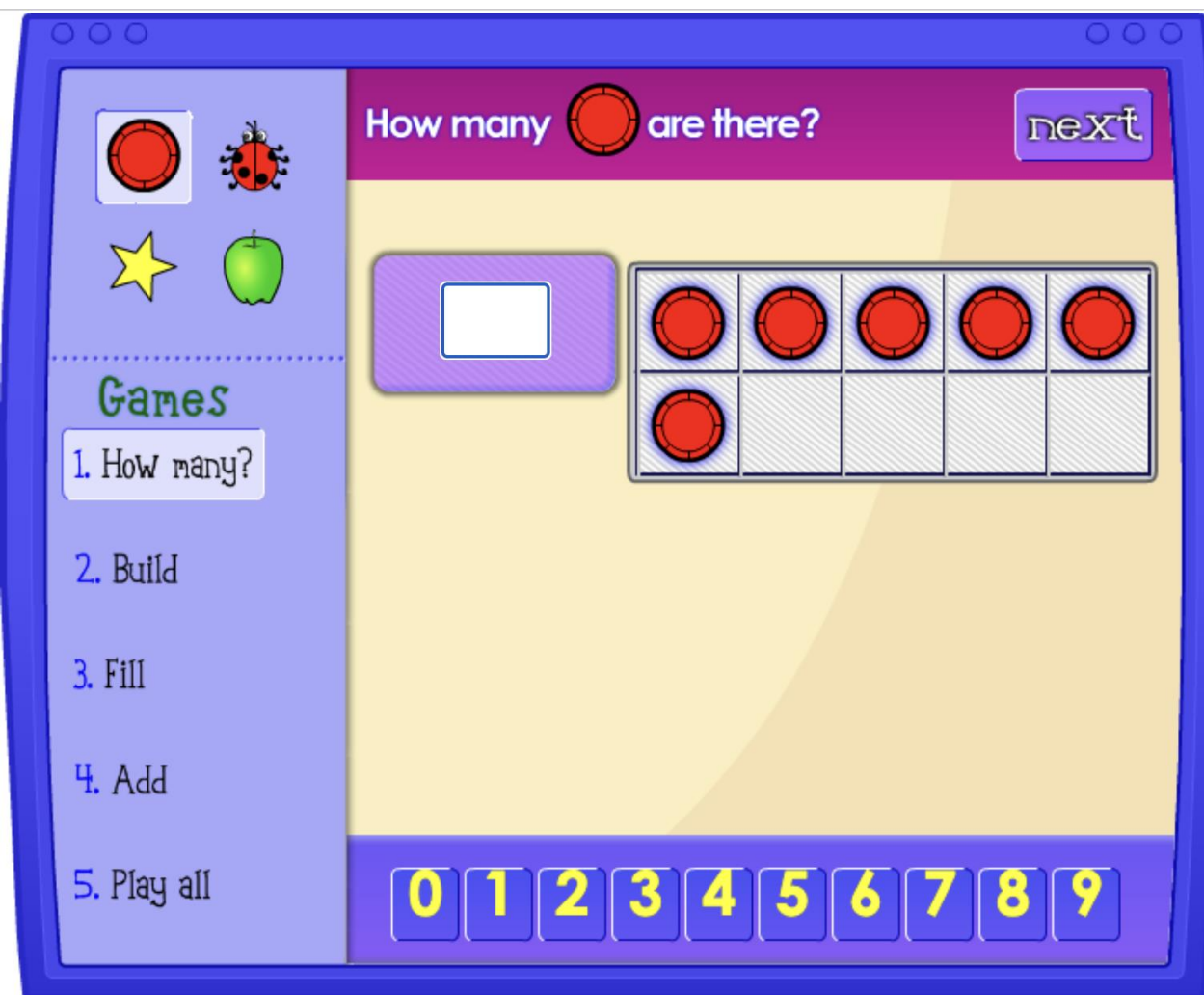
Metacognitive Strategy Instruction



<https://toytheater.com/two-color-counter-ten-frame/#>



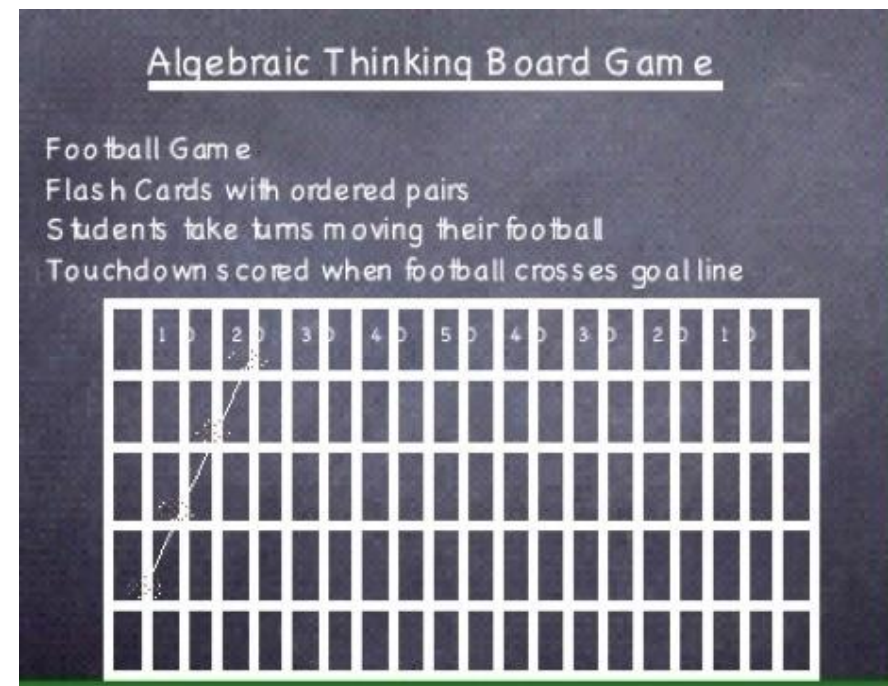
<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/>



Research Supported Practices



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Board Games



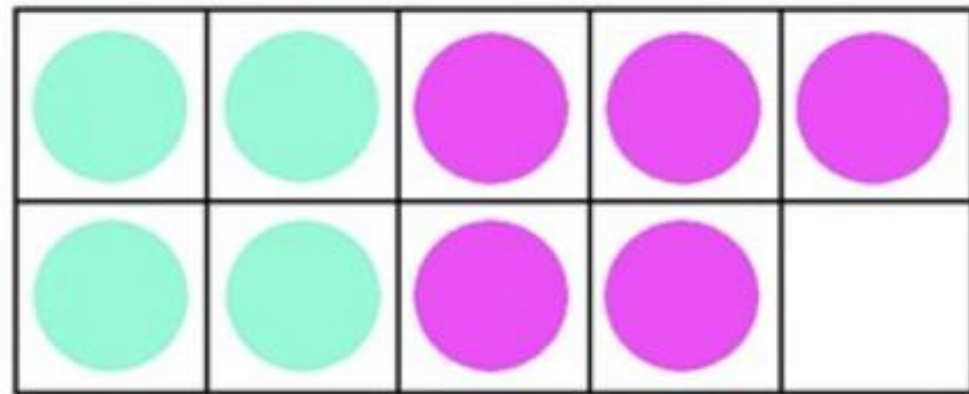
Game-Based Apps

Research Supported Practices

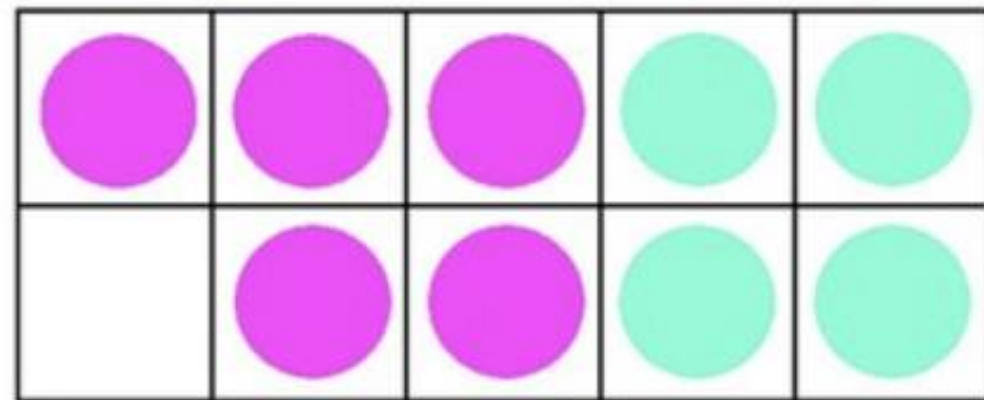


- Peer Tutoring/Structured Cooperative Groups
- Authentic Contexts
- Purposeful Use of Technology
- Game-Based Learning**
- Structured Language Experiences
- Visuals
- Explicit Systematic Instruction
- Metacognitive Strategy Instruction

We have already introduced many visuals-online, pictures, & manipulatives. Now it's time to consider how to make intentional and direct connections between the visuals and the numbers/fact families. The key is to be very specific and explicit about the connections.



$$4 + 5 = 9$$

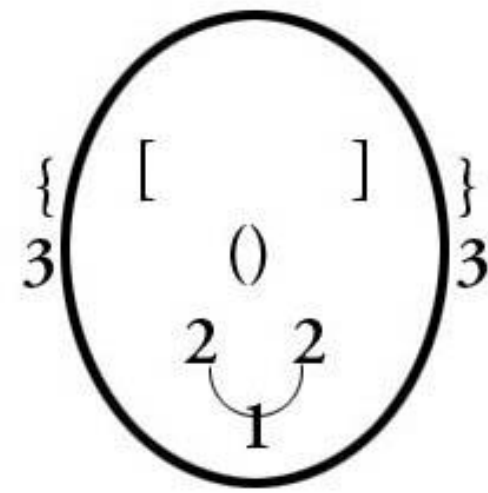
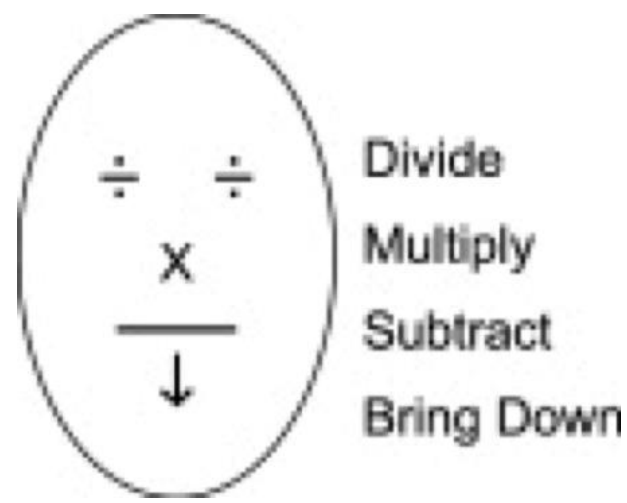


$$5 + 4 = 9$$

Research Supported Practices



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- Visuals**
- Explicit Systematic Instruction**
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Discover the sign.

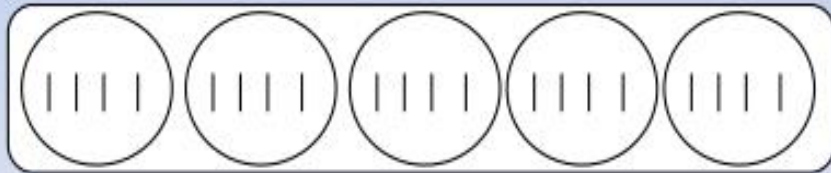
5 (x) 4 =

The DRAW Strategy for Basic Operations

Read the problem.

"Five times four equals..." or "Five groups of four is..."

AnsWER the problem, or draw and check.



Write the answer.

5 x 4 = 20

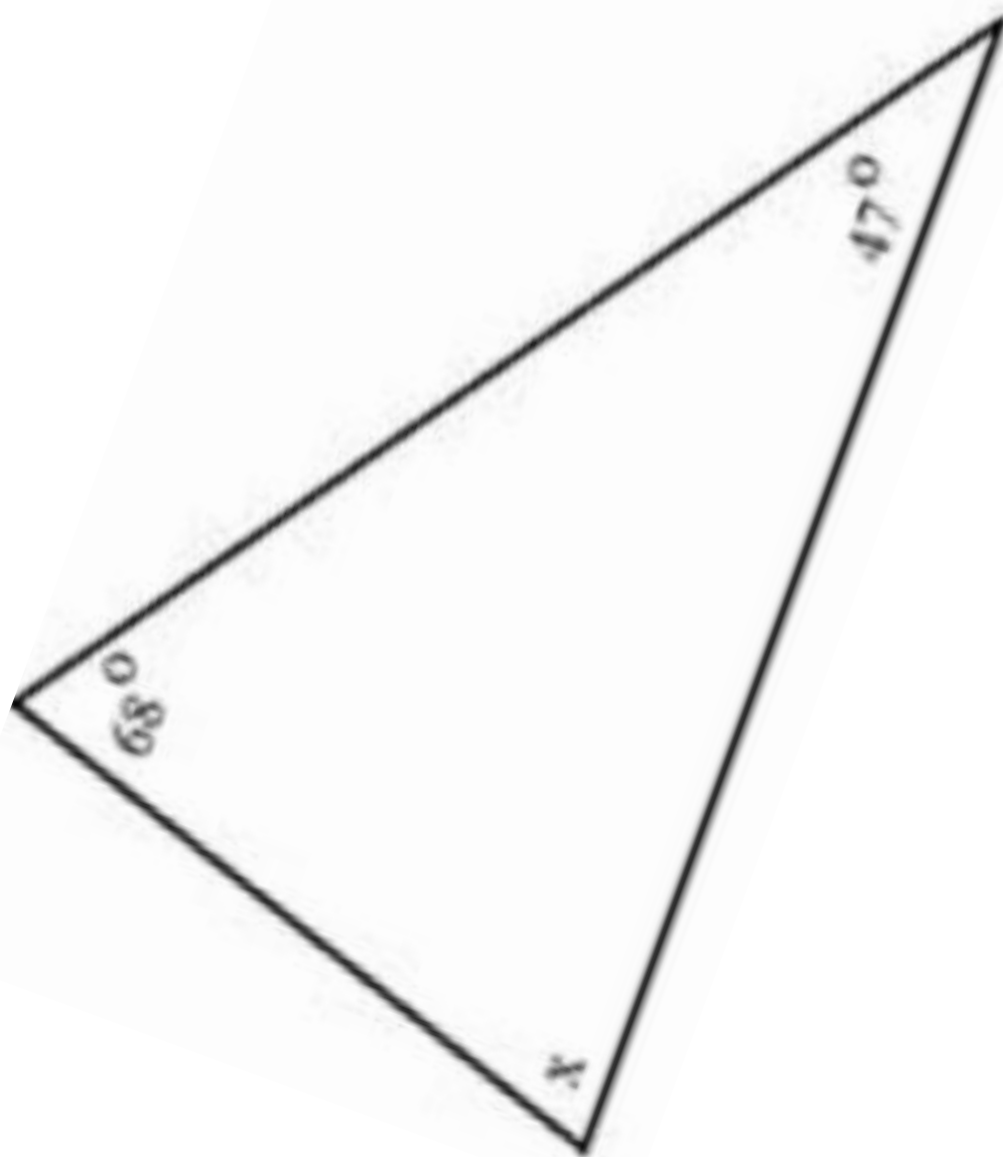
STRATEGY SHEET	
Paste problem here.	
I solved the problem by:	
	Counting Chips or Base Ten Materials
	Counting Up
	Make a Ten
	Other Strategy
$\underline{\quad} + \underline{\quad} = \square$	

Research Supported Practices



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- Authentic Contexts
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- Metacognitive Strategy Instruction

Middle/High School: Algebraic Equations (*By Leveraging Geometry*)



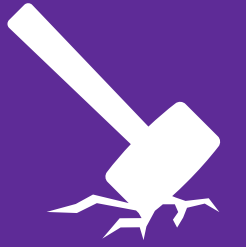
This is a typical middle/high school level math problem of finding the missing angle in a triangle. Typically, teachers might “tell” students that the three angles equal 180, and then use the “I do, we do, you do” approach to having students complete MANY of the same algebraic problems of solving for “x.”

We suggest a different approach that enriches the mathematics experiences for the middle/high school student with DS.



We'll start by assessing the related measurement & geometry foundation

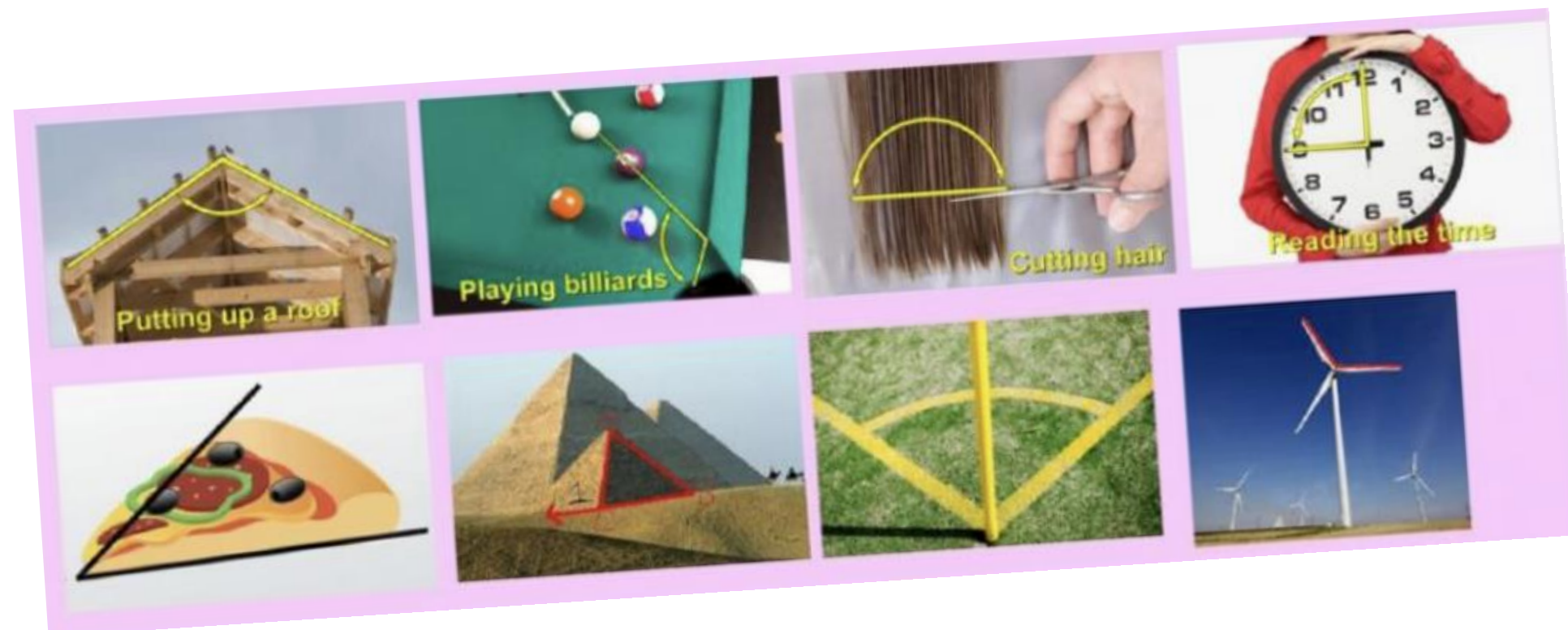
Instructional Focus



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- Engage in the Mathematics Processes

Coherent & Connected Content: Angles

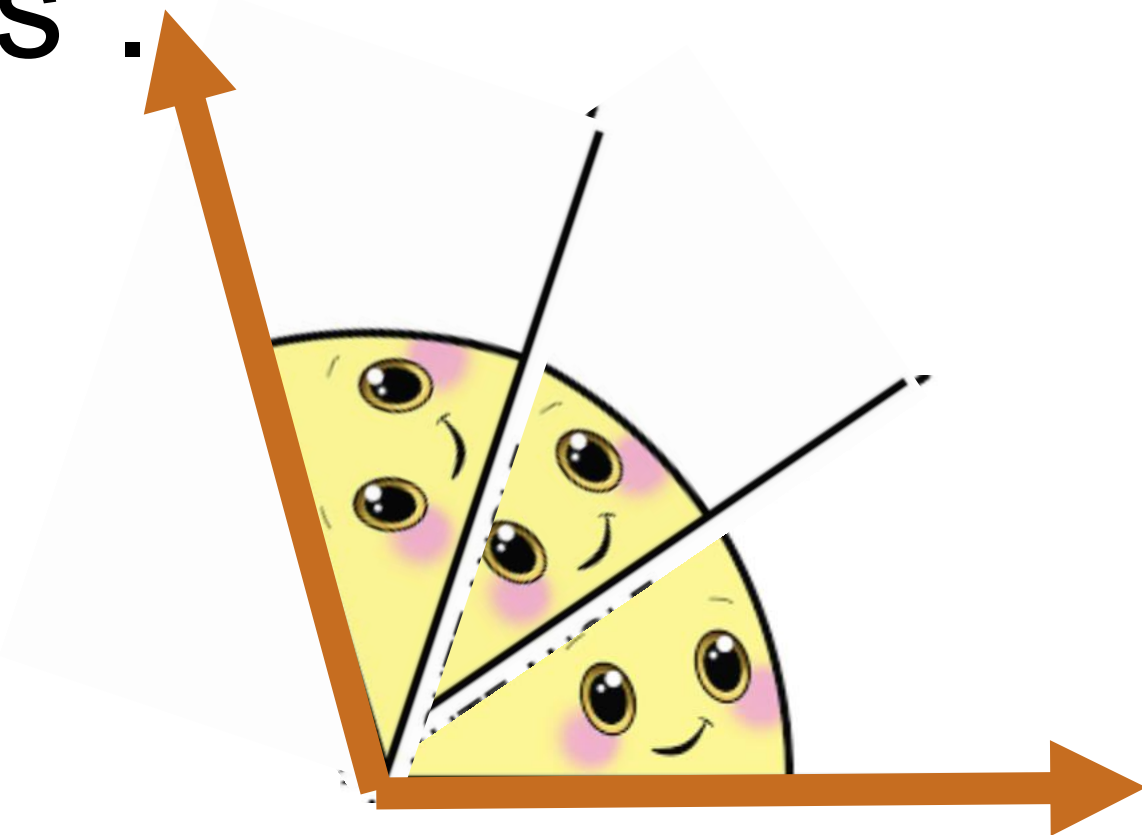
- Exploration of the attribute of an angle. Students go on a scavenger hunt throughout the classroom identifying larger and smaller angles.



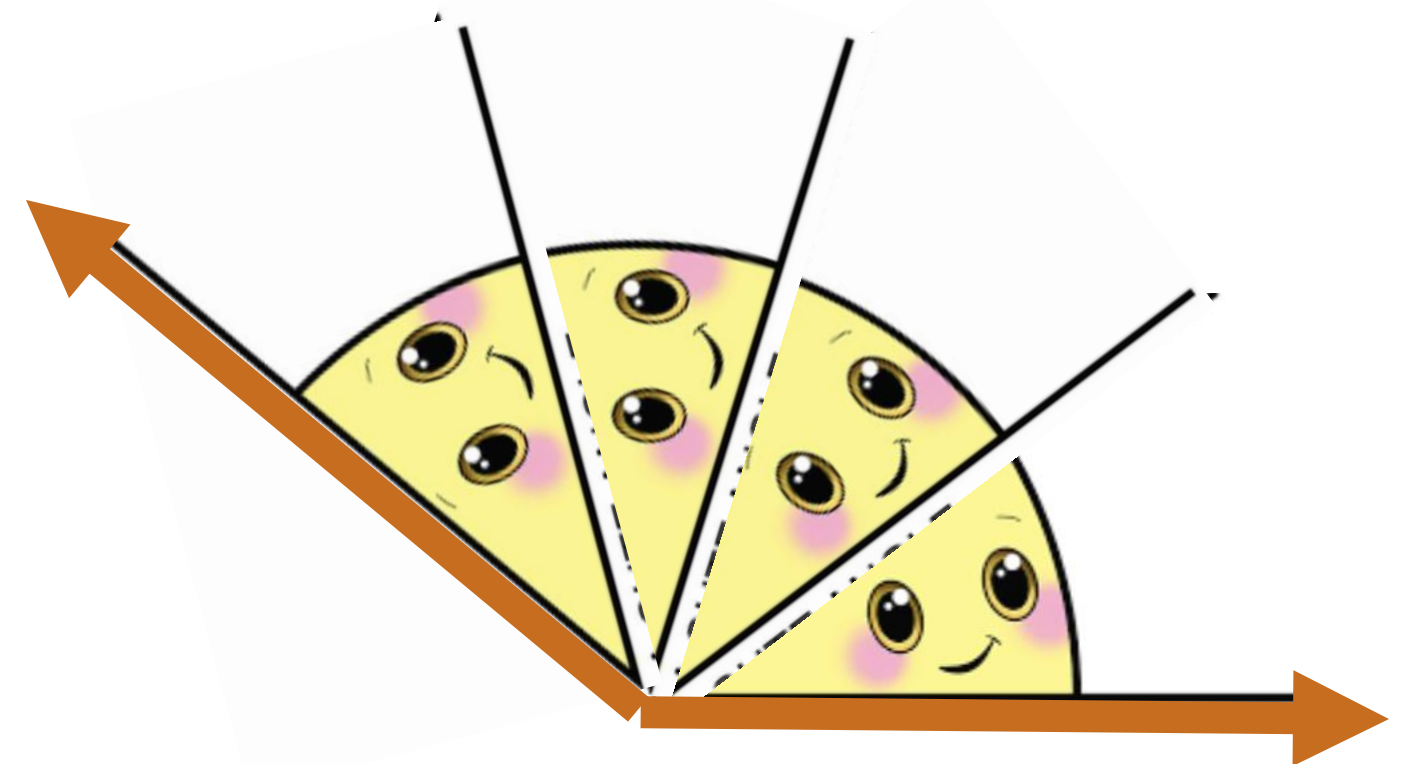
Coherent & Connected Content: Angles

- Then use a “non-standard unit” angle and have students measure other angles with this base angle:

- 3 “units”.

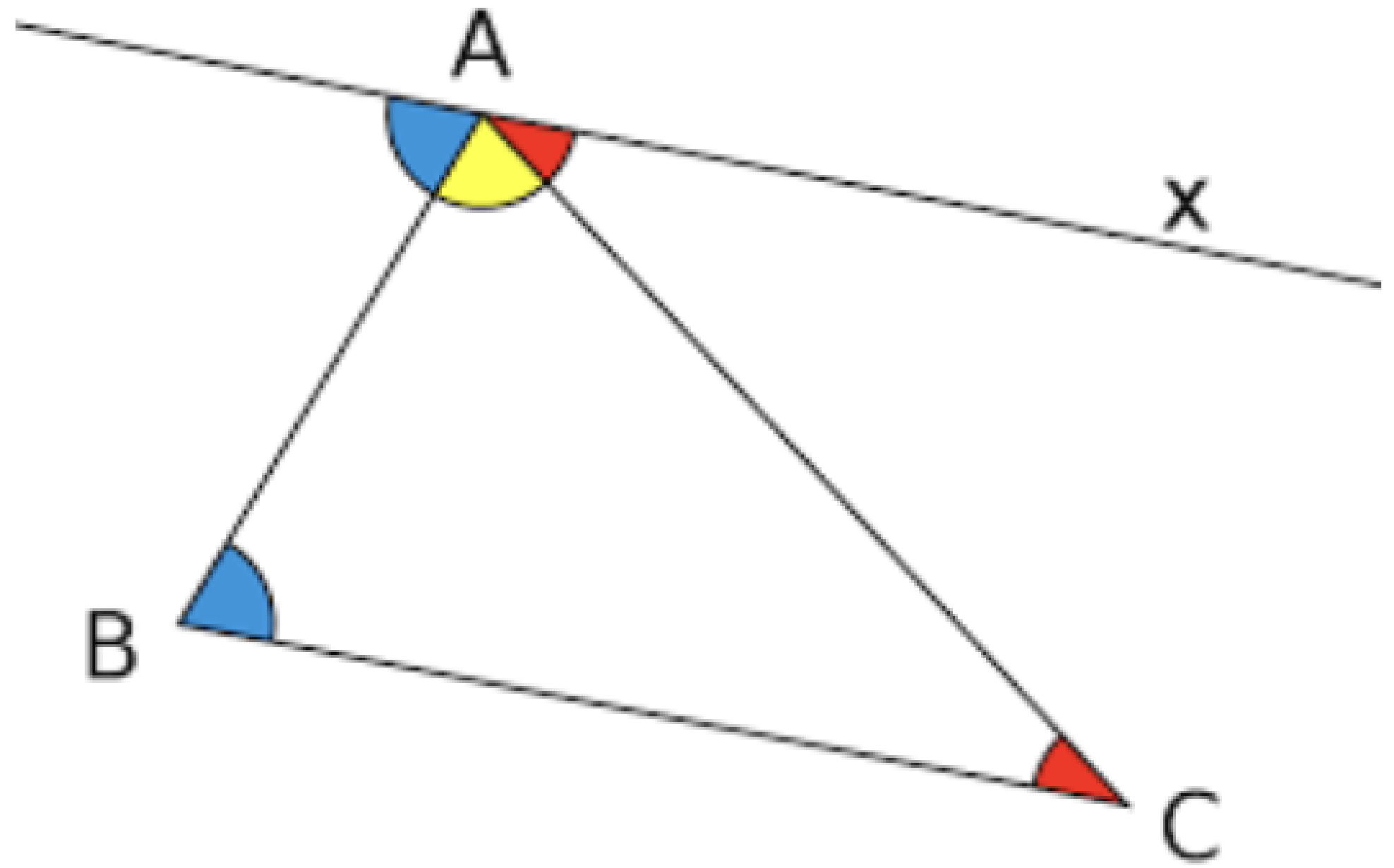


- 4 “units



Coherent & Connected Content: Angles

- Introduce the concept of degrees, the 360° circle, the 180° line and then explore the degrees in a triangle.



Areas of Difficulty



- Motor Skills (Fine)
- Expressive Language
- Verbal/auditory Short-term Memory
- Number Skills
- Other _____



_____ **Turn & Talk**

Areas of Strength



- Social Interaction
- Empathy
- Visual Inputs & Short-term Memory
- Self-help Daily Living Skills
- Word Reading/Vocabulary Acquisition
- Technology Oriented
- Other _____

Research Supported Practices



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